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Forced Vibration Analysis of Breathing Cracked Cantilever Beam using MATLAB

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Abstract— Cantilever beam with breathing crack is selected and forced vibration analysis is performed in MATLAB. The cantilever beam is converted into the equivalent single degree of freedom system. The simple single-degree-of-freedom system with time varying stiffness is employed to simulate the dynamic behavior of the beam. The equation of motion of breathing cracked cantilever beam with time varying stiffness is developed and solved using MATLAB. The frequency, amplitude and acceleration of cracked and uncracked beam response is determined. Analysis is carried out in both time and frequency domains, which is aimed to identify the dynamic response associated with the existence of breathing crack.

Index Terms— Cantilever Beam, Breathing crack, Forced vibration, MAT LAB, Time domain, Frequency domain, Stiffness.

1 Introduction

Surface cracks occur frequently in the structures of engineering applications. The effect of these cracks on the performance of the structure is more severe. Identification of crack depths and location of reference point are the standard methods in performance monitoring of the structures. Currently available non-destructive testing (NDT) methods, such as acoustic, ultrasonic and magnetic field methods are time consuming. Vibration analysis based technique has been proved fast and inexpensive for crack identification. The vibration analysis of a physical system consists of four major steps, namely mathematical modeling of a physical system, formulation of the governing equations, mathematical solution of the governing equation, and physical interpretation of the results. E. Douka et al [1&2] has investigated the dynamic behavior of a cantilever beam with a breathing crack both theoretically and experimentally. S.M.Cheng et al [3] proposed that the natural frequency reduction for a breathing crack is much smaller than for an open crack. Jyoti K. Sinha et al [4] has studied the experimental vibration behavior of a free–free beam with a breathing crack is simulated for a sinusoidal input force using a simple FE model for a crack in the beam. The fatigue crack was introduced in the form of breathing crack and the model which opens when the normal strain near the crack tip is positive, otherwise it closes [5]. The nonlinear behavior was found on time history and frequency spectrum for each vibration mode[6]. The changes in the dynamic behavior of cracked structures can be used to deduce the size and location of the crack [6-8]. O. N. L. Abraham et al [9] was investigated a method which utilizes substructure normal modes to predict the vibration properties of a cantilever beam with a breathing transverse crack. Crack identification in structures has been the subject of intensive investigations since the last five decades. Many studies have been carried out in an attempt to find methods for non-destructive crack detection in structural members. However, a consistent cracked beam vibration theory is yet to be developed. There are still many unanswered questions, especially in the area of closing cracks in beams.

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2 Modelling Of Breathing Crack

In this paper, a simple breathing crack model is developed for the cracked cantilever beam. To make an analysis of the dynamic behavior of a cracked beam vibrating at its first mode is analyzed. In the analysis, a cantilever beam with a breathing crack is considered as shown in Fig. 1(a). The cantilever beam is converted into the equivalent single degree of freedom (SDOF) model as shown in Fig. 1(b). The beam is excited by a sinusoidal force causing the crack to open and close. Based on the assumption that the beam vibrates at its fundamental mode, the time-varying stiffness of the beam can be modeled using a simple periodic function of time.

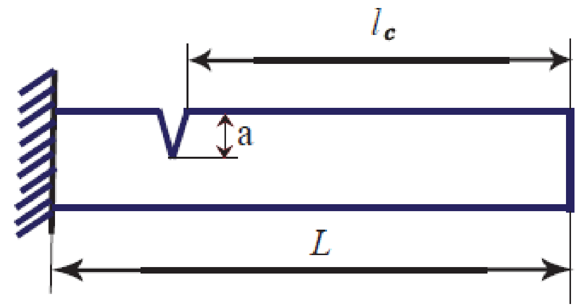

Fig. 1(a). Geometry of the cracked cantilever beam

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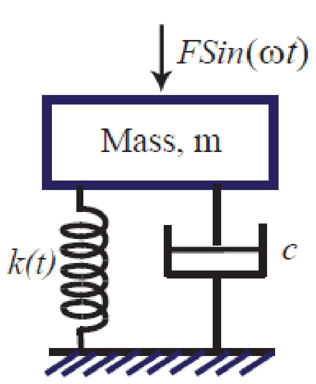

Fig. 1(b). Equivalent SDOF model

The load-displacement response of a breathing crack can be represented by the curve shown in Fig: 2.

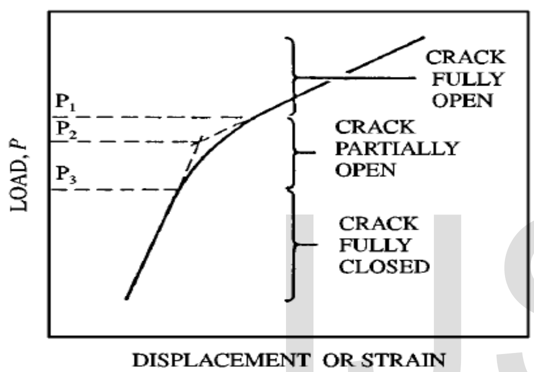

Fig. 2. Schematic load-displacement curve

The points P_1, P_2 and P_3 are shows that crack is fully open, partially open and fully closed respectively. As the stiffness 'K' of the structure is a measured of the resistance of load of an elastic body to deformation. For an elastic body with by an elastic body with a single degree of freedom system (SDOF) the stiffness are given as, K=F/δ

Where F is the force applied on the body and δ is the displacement. Here, time (t) is chosen as the independent variable because the state of crack opening depends on the level of load, which varies with time due to vibration.

The dynamic response of a breathing crack at its first mode in a single-degree-of-freedom system, the stiffness may be expressed as

$$k(t) = k_0 + k_{\Delta c}(1+\cos\omega t) = k_1 + k_2$$

Where ω is the crack breathing frequency, it is equal to the excitation frequency.

$k_1 = k_0$ is the stiffness of the structure when the crack is fully open. And the amplitude of the stiffness change is given by

$$k_{\Delta c} = 1/2\,(k_c - k_0)$$

Where k_c is the stiffness when the crack is closed, hence the stiffness change is $k_2 = k_{\Delta c}(1+\cos\omega t)$

The above stiffness model assumes that the crack is completely closed when ωt = 2nπ, Where n = 1, 2, 3…n is any integer. Then $k(t) = k_c$

When $\omega t = (2n-1)\Pi$, Where n = 1, 2, 3…n is any integer.

Then $k(t) = k_0$

The crack is in the fully open state. Otherwise the crack is in a state of partial closure.

The present model simulates the change of the structural stiffness as a continuous function of time, i.e., when the crack opens and closes at a rate of ω. The coefficients k_0 and k_c are determined from the stiffness properties of the structure when the crack is completely open and completely closed respectively.

When the crack is completely closed, the structure acts as one without a crack, and the stiffness k_c is determined using structural mechanics methods. When the crack is completely open, the stiffness k_0 can be determined using fracture mechanics.

Incorporating the breathing crack model into a single-degree-of-freedom system, the governing equation for forced vibration can be expressed as [1]

$$m\ddot{u} + c\dot{u} + [k_0 + k_{\Delta c}(1+\cos\omega t)]u = F\sin\omega t$$

The generalized stiffness k_c of the un-cracked beam is given by:

$$k_c = \frac{1}{c} = \frac{EI\pi^4}{32L^3}$$

The stiffness when the crack is open $k_0 = 1/c_0$

The total flexibility $c_0 = c + \Delta c$

Where Δc is the change in the flexibility due to the presence of a crack.

The change in the flexibility of a cracked beam can be derived from the equation developed by Dimarogonas and Paipetis as [1]

$$\phi = 19.6\frac{a^{10}}{b^8} - 40.69\frac{a^9}{b^7} + 47.04\frac{a^8}{b^6} - 32.99\frac{a^7}{b^5} + 20.3\frac{a^6}{b^4}$$

$$-9.98\frac{a^5}{b^3} + 4.6\frac{a^4}{b^2} - 1.05\frac{a^3}{b} + 0.63a^2$$

Hence, the total flexibility of the beam containing an open crack is given by

$$c_0 = \Delta c + c_{nocrack}$$

Where $C_{\text{no crack}}$ is the beam's flexibility without a crack.

The differential equation of motion is solved using MATLAB. The whole analysis is carried out in both time and frequency domains.

3 Numerical Analysis

Numerical simulations were performed considering a Plexiglas beam of: Total length L= 230 mm and Rectangular cross section bXd= 20X20 mm². A crack of varying depth is introduced at l_c= 0.9L mm from the free end.

For the beam material:

Young's modulus E =2.5X10³ N/mm²,

Density of ρ = 1200X 10⁻⁹ kg/mm²,

Poisson ratio of ϑ= 0.31, and

Damping factor c=0.15 is used.

A harmonic force of amplitude F=10N is assumed in all cases.

A generalized mass of the beam is m=0.228 m'L.

The forcing frequency "ω" is assumed to be equal to

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half of the first radian frequency *f* of the undamaged beam. In the previous studies it has been shown when the excitation frequency is approximately half the natural frequency of the beam, the non-linear behavior of a beam with a breathing crack is most clear. For the beam considered, the first natural frequency is f_1=91 HZ and $\omega_1 = 2\pi f_1$ = 571.7698 rad/Sec

Therefore forcing frequency $\omega = {\omega_1}/{2}$

The equation of motion is integrated and the dynamic response of the beam is obtain the initial conditions were initial velocity u'(0)=0 mm/s and u(0)=0.05 mm, the whole analysis is implemented using MATLAB.

3.1 Time Domain Analysis of Uncracked Cantilever Beam

The time domain is a record of the response of a dynamic system, as indicated by some measured parameter, as a function of time. The time histories of the calculated acceleration are presented in Fig.3. The figure shows pure sinusoidal waveform corresponds to the response of the uncracked beam.

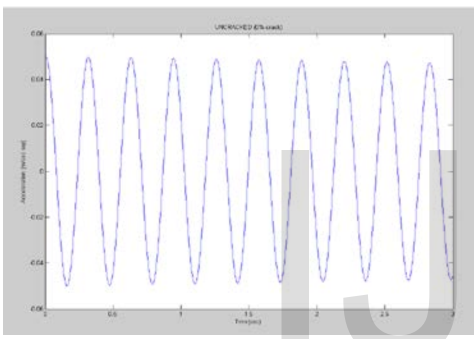

Fig. 3. Vertical Displacement of the Uncracked Cantile-

3.2 Time Domain Analysis of Cracked Cantilever Beam

Fig 4 to 7 shows the time domain plot of 10%, 20%, 30% and 40% crack depth ratios and crack position at 207 mm from fixed end. Crack depth ratio is a ratio of crack depth to depth of the beam. The acceleration (g) values are increasing when crack depth increases. The stiffness of cracked beam are decreasing with cracked depth increases. The crack depth is increased harmonic distortion in sinusoidal wave has also increased.

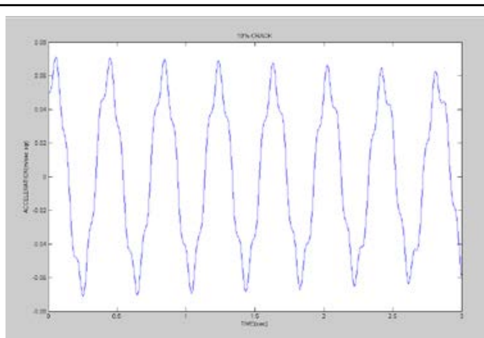

Fig. 4. 10% Crack depth ratio

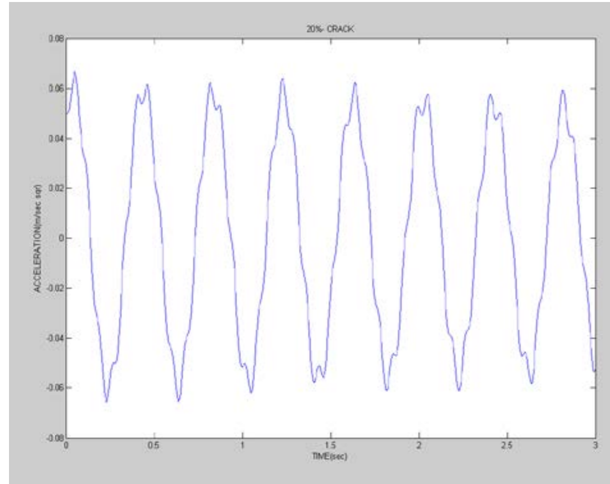

Fig. 5. 20% Crack depth ratio

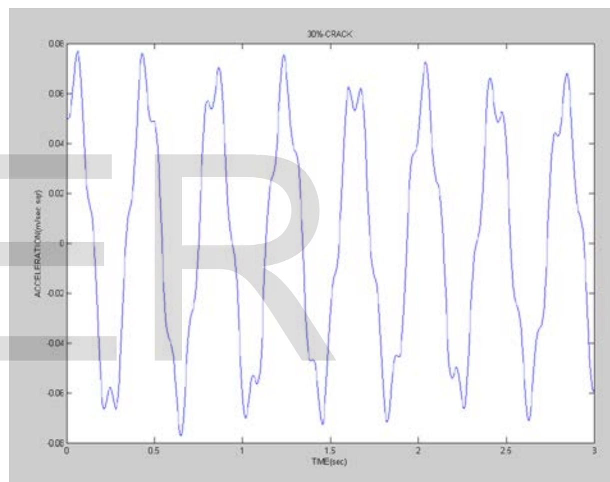

Fig. 6. 30% Cracked depth ratio

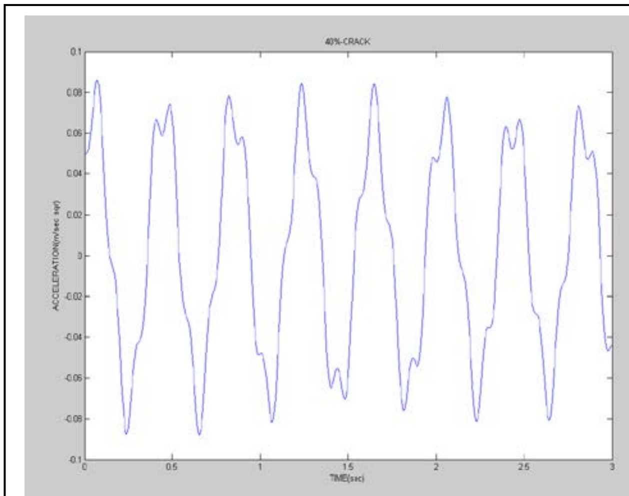

Fig. 7. 40% Cracked depth ratio

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3.3 Frequency Domain of Uncracked Cantilever Beam

The frequency domain is analysis of mathematical function or signals with respect to frequency rather than time. It shows the magnitude of discrete Fourier transform of the uncracked beam with one component at the forcing frequency $f_c/2$. It presents the spectrum of the measured response of the test beam. It can be observed that the beam vibrates predominantly at its fundamental frequency. The peak in frequency domain represents as resonant frequency as shown in Fig.8.

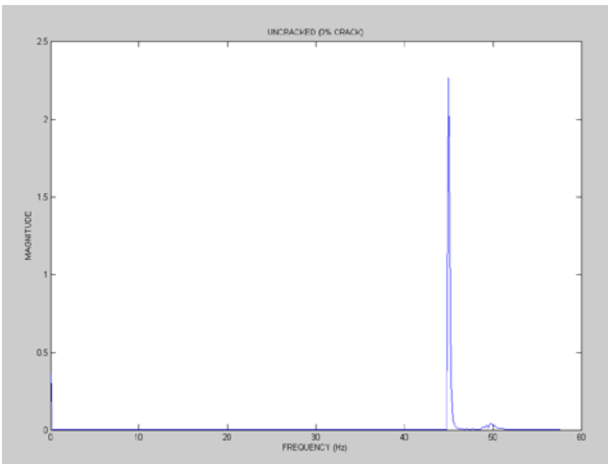

Fig. 8. Vertical Displacement of the Uncracked Cantilever Beam Magnitude vs. Frequency

3.4 Frequency Domain of Cracked Cantilever Beam

Fig 9 to 12 shows the frequency domain plot of 10%, 20%, 30% and 40% crack depth ratios and crack position at 207 mm from fixed end. It exhibits peaks at the forcing frequency and its first harmonic indicating the non-linearity induced by the presence of the breathing crack. The highest resonant frequency values increased when crack depth increases.

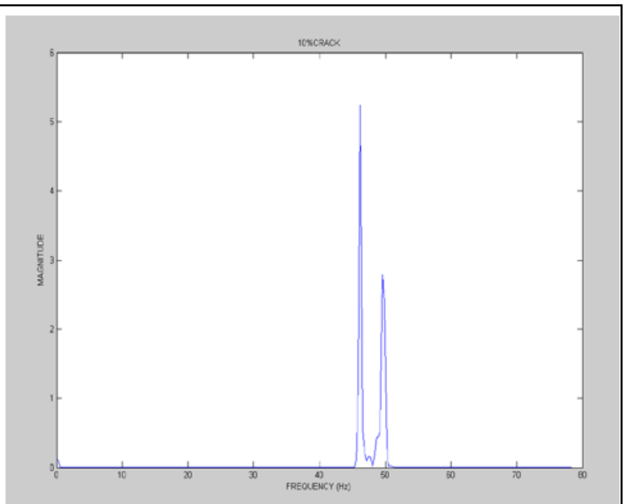

Fig. 9. 10% Crack Depth Ratio

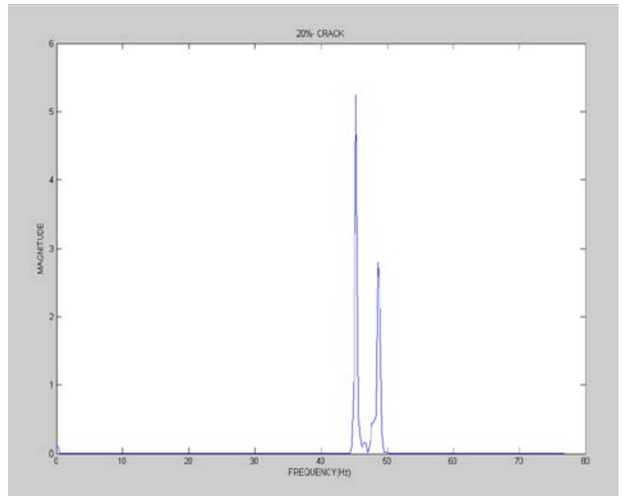

Fig. 10. 20% Crack depth ratio

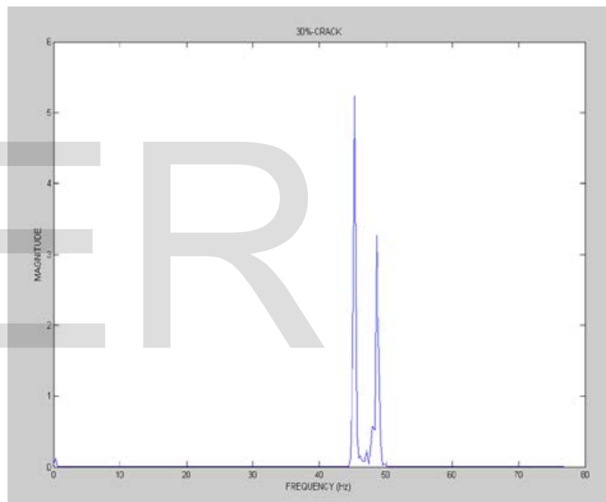

Fig. 11. 30% Crack Depth Ratio

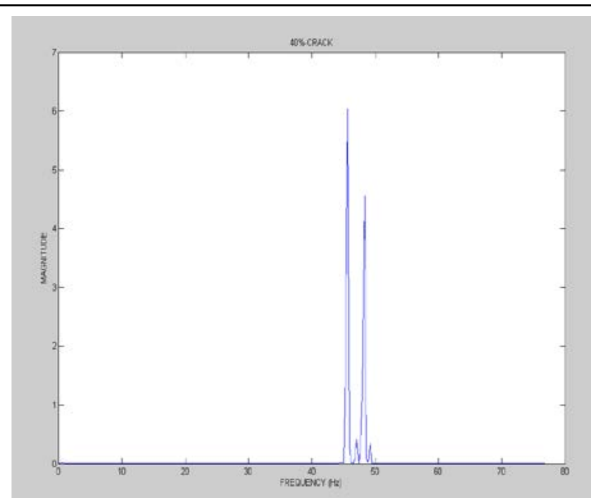

Fig. 12. 40% Crack Depth Ratio

4 Conclusion

The equation of motion of a cracked cantilever beam is developed. The equation of motion is integrated by using MATLAB with the initial conditions. The results are shown in both time domain and frequency domain. The uncracked beam shows pure sinusoidal wave in time domain. The breathing crack model shows the deviation in a pure sinusoidal wave. The crack depth is increased harmonic distortion in sinusoidal wave has also increased. The frequency domain shows the frequency, magnitude of the uncracked beam with one component that equals to forcing frequency. It can be observed that the beam vibrates predominantly at its fundamental frequency. The frequency domain shows the frequency, magnitude of the cracked beam with two components because of non-linearity of the crack. The depth of the crack increases the height side bands increases, which show more severe damage of the beam.

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